Proceedings Trim Size: 9.75in x 6.5in
Text Area: 8in (include runningheads) x 5in
Main Text is 10/13pt

For Half-Title Page (prepared by publisher)

Publishers' page — (Blank page)

For Full Title Page (prepared by publisher)

For Copyright Page (prepared by publisher)

PREFACE

The *Tenth International Conference on Recent Progress in Many-Body Theories* (RPMBT-10) was held at the University of Washington in Seattle, USA during the period 10-15 September, 1999. The present volume contains the texts of most of the invited talks delivered at the conference and a selection from among the many poster presentations.

The general format and style of the conference followed the accepted and well-developed pattern for the series, focusing on the development, refinement and important applications of the techniques of quantum many-body theory. The intention of the series has always been to cover in a broad and balanced fashion both the entire spectrum of theoretical tools developed to tackle the quantum many-body problem and their major fields of application. One of the main aims of the series is to foster the exchange of ideas and techniques among physicists working in such diverse areas of applications of many-body techniques as nuclear and subnuclear physics, astrophysics, atomic and molecular physics, quantum chemistry, complex systems, quantum field theory, strongly correlated electronic systems, magnetism, quantum fluids and condensed matter physics.

Quantum many-body theory as a discipline in its own right dates largely from the 1950's, and is hence in many senses already a mature subject. Despite this apparent maturity the field remains vibrant and active, vigorous and exciting, vital and important. Indeed, the successes, importance and vitality of the field have very clearly been recognized by the sharing of the 1998 Nobel Prizes in both Physics and Chemistry by the many-body theorists Robert Laughlin, Walter Kohn and John Pople. It is a source of great pleasure to all of us who work in quantum many-body theory that important achievements in our subject have been thus recognized at the very highest level. We were also especially delighted that two of these then most recent Nobel Laureates, Kohn and Laughlin, accepted invitations to deliver keynote lectures at RPMBT-10...

A major challenge will be to define and explore these important areas of overlap in such likely arenas as phase transitions and the related competing effects of quantum versus thermal fluctuations; spontaneous symmetry breaking; the unifying role of gauge invariance (and perhaps other symmetries); concepts from complexity theory, including the role of correlations and fluctuations in complex systems, and the competition between analysis and synthesis as modes of understanding many-body systems; entropy and its generation and flow; and nonlinearity and its appearance and role at the boundary between microscopic and mesoscopic descriptions of many-body systems. One can perhaps begin to glimpse some insights within the present proceedings into some of these as areas of future forefront activity in quantum many-body theory. In this case it may well be that RPMBT-10 and its immediate successors will later be identified as the first in the series at which the seeds of the new discourse between quantum many-body/field theory and statistical physics took root.

In any event, the Local Organizing and Programme Committees deserve great thanks in creating a well-run and productive meeting, with an exciting programme of talks and poster presentations. It is a pleasure to thank all of them for their hard work, especially Aurel Bulgac who, as Chairman, has led and guided them throughout.

R.F. Bishop
(Chairman, International Advisory Committee for the Series of International Conferences on Recent Progress in Many-Body Theories)

Manchester, U.K.
31 December 1999

ORGANIZING COMMITTEES

SERIES EDITORIAL BOARD

for the Series on Advances in Quantum Many-Body Theory

R. F. Bishop (Chairman)	– UMIST, Manchester, UK
C. E. Campbell	– University of Minnesota, USA
J. W. Clark	– Washington University, St. Louis, USA
S. Fantoni	– SISSA, Trieste, Italy

INTERNATIONAL ADVISORY COMMITTEE

for the Series of International Conferences on
Recent Progress in Many-Body Theories

R. F. Bishop (Chairman)	– UMIST, Manchester, UK
S. Fantoni (Secretary)	– SISSA, Trieste, Italy
C. E. Campbell (Treasurer)	– University of Minnesota, USA
H. Kümmel (Hon. President)	– Ruhr-Universität Bochum, Germany
J. A. Carlson	– Los Alamos National Laboratory, USA
S. A. Chin	– Texas A & M University, USA
J. W. Clark	– Washington University, St. Louis, USA
P. Fulde	– Max-Planck-Institut für Komplexer Systeme, Dresden, Germany
A. Kallio	– University of Oulu, Finland
E. Krotscheck	– Johannes Universität Linz, Austria
C. Lhuillier	– Université Pierre et Marie Curie, Paris
A. MacDonald	– Indiana University, Bloomington, USA
E. Manousakis	– Florida State University, USA
D. Neilson	– University of New South Wales, Sydney, Australia
M. L. Ristig	– Universität zu Köln, Germany

CONTENTS

FOR PROCEEDINGS EDITORS:
COMBINING CONTRIBUTIONS USING WS-PROCS975x65 MASTER DOCUMENT IN LATEX2e

First Author

World Scientific Publishing Co, Production Department,
5 Toh Tuck Link, Singapore 596224,
E-mail: wspc@wspc.com.sg

ws-procs975x65.tex is the master file to input all the papers from front matter, body and end matter files.

Keywords: Master file; LATEX; Proceedings; World Scientific Publishing.

1. Master File

You can obtain these files from the following website:
`http://www.wspc.com.sg/style/proceedings_style.shtml`,
`http://www.icpress.co.uk/authors/stylefiles.shtml#proceedings` and
`http://eproceedings.worldscinet.com/pro_editors.shtml`.

All the subdocuments are arranged in this master file in the following sequence:

```
\documentclass{ws-procs975x65}
\usepackage{ws-toc,ws-multind}
\makeindex{author}                   %to invoke author index
\begin{document}
\titlepages
\input preface.tex
\cleardoublepage
\include organizers.tex
\mastertoc
%\part{Part Title}{}                 %optional part title
\include{proc1}
\include{proc2}
%\part{Second Part}{}                %optional part title
\include{proc3}
\printindex{author}{AUTHOR INDEX}    %to print Author Index
\end{document}
```

2. Pagination

Pages i–iv are always the same and are prepared by the publisher. The pagination of the rest of the front matter depends on the length of the various sections (preface, organizing committee's text, contents). Each front matter section should start on an odd right-hand page. The following is an example of proceedings pagination:

(i) Front matter:

page i	—	half-title page, prepared by publisher
page ii	—	blank page
page iii	—	full-title page, prepared by publisher
page iv	—	copyright page, prepared by publisher
pages v–ix	—	preface text
page xi	—	organizing committees text
pages xiii–xx	—	contents text (TOC)

The first available front matter page is page v.

(ii) Body text:

odd page — First article, followed by remaining articles

(iii) Back matter (optional):

odd page — participants list

Back matter text must start on an odd page.

If the organization of the contributors' manuscripts is different from the above pagination guidelines, please e-mail the respective desk-editor for advice.

Table 1. This table shows how the author names should appear in running head and TOC depending upon the number of authors contributing that paper.

No. of Authors	Author Names
1	L. Hatcher
2	I. A. Pedrosa & I. Guedes
3	B. Feng, X. Gong & X. Wang
4 and more	S. R. Choudhury *et al.*

Note: For TOC and Running Heads, the author names should appear in initial and surname format, e.g. LEE HATCHER should be abbreviated as L. HATCHER.

3. Running Head

Preparation of the running head is optional. Each contribution must have `\markboth` at the preamble to set the running head.

```
\markboth{Author's Name on Even Page}{Article Title on Odd Page}
```

4. Master TOC

Each contribution must have `\wstoc{#1}{#2}` at the preamble to create the combined table of contents.

```
\wstoc{Article Title}{Author's Name}
```

5. Preparing the Individual Contributions for Combining

The following highlighted changes should be made in all the contributions provided by the individual contributors before including them in the master document.

Contribution provided by an individual contributor:

```
%proc1.tex
\documentclass{ws-procs975x65}
\begin{document}
\title{PAPER TITLE}
\author{A. AUTHOR}

\address{Institute of ...}
\begin{abstract}
We search for ...
\end{abstract}
\keywords{Keyword1; ...}
\bodymatter
\section{Introduction}
String theory ...
\begin{thebibliography}{00}
\bibitem{t1} A. Sen, ...
\bibitem{t2} P. Horava, ...
...
\end{thebibliography}
%for BiBTeX users
%\bibliographystyle{ws-pro...}
%\bibliography{sample}
\end{document}
```

Modified contribution, ready to get included in the master document:

```
%proc1.tex
\markboth{A. Author}{Paper Title}
\wstoc{Paper Title}{A. Author}
\title{PAPER TITLE}
\author{A. AUTHOR}
\aindx{Author, A.}
\address{Institute of ...}
\begin{abstract}
We search for ...
\end{abstract}
\keywords{Keyword1; ...}
\bodymatter
\section{Introduction}
String theory ...
\begin{thebibliography}{00}
\bibitem{t1} A. Sen, ...
\bibitem{t2} P. Horava, ...
...
\end{thebibliography}
%for BiBTeX users
%\bibliographystyle{ws-pro...}
%\bibliography{sample}
\vfill
\pagebreak
```

6. Author Index

To create an "author" index, the following declarations should be included in the master TEX file:

```
\makeindex{author}      % in the preamble to invoke author index
\printindex{author}{AUTHOR INDEX} % at end to print author index
```

In text, the author index entries are marked with:
`\index{autor}{entry}` or `\aindx{entry}`

7. Compiling the Master File in LATEX2e

To complete the job, compile your file as follows:

(1) latex ws-procs975x65
(2) latex ws-procs975x65
(3) bibtex proc2 `% Chapters using BIBTEX database should`
`% be compiled individually in 'bibtex'.`
(4) makeindex author `% author index`
(5) latex ws-procs975x65
(6) latex ws-procs975x65

References

1. L. Lamport, *LATEX, A Document Preparation System.* (Addison-Wesley, Reading, MA, 1994), 2nd edition.
2. L. Lamport, *Make Index: An Index Processor For LaTeX*, (1987).

FOR PROCEEDINGS CONTRIBUTORS: USING WORLD SCIENTIFIC'S WS-PROCS975X65 DOCUMENT CLASS WITH LATEX2e

A. B. AUTHOR* and C. D. AUTHOR

University Department, University Name,
City, State ZIP/Zone, Country
**E-mail: ab_author@university.com*
www.university_name.edu

A. N. AUTHOR

Group, Laboratory, Street,
City, State ZIP/Zone, Country
E-mail: an_author@laboratory.com

This article explains how to use World Scientific's ws-procs975x65 document class written in LATEX2e. This article was typeset using ws-procs975x65.cls and may be used as a template for your contribution.

Keywords: Style file; LATEX; Proceedings; World Scientific Publishing.

1. Using Other Packages

The class file loads the packages `amsfonts, amsmath, amssymb, chapterbib, cite, dcolumn, epsfig, rotating` and `url` at startup. Please try to limit your use of additional packages as they often introduce incompatibilities. This problem is not specific to the WSPC styles; it is a general LATEX problem. Check this article to see whether the required functionality is already provided by the WSPC class file. If you do need additional packages, send them along with the paper. In general, you should use standard LATEX commands as much as possible.

2. Layout

In order to facilitate our processing of your article, please give easily identifiable structure to the various parts of the text by making use of the usual LATEX commands or by using your own commands defined in the preamble, rather than by using explicit layout commands, such as `\hspace, \vspace, \large, \centering`, etc. Also, do not redefine the page-layout parameters. For more information on layout and font specifications, please refer to our `Layout and Font Specification Guide`.

3. User Defined Macros

User defined macros should be placed in the preamble of the article, and not at any other place in the document. Such private definitions, i.e. definitions made using the commands `\newcommand, \renewcommand, \newenvironment` or `\renewenvironment`, should be used with great care. Sensible, restricted usage of private definitions is encouraged. Large macro packages and definitions that are not used in this example article should be avoided. Please do not change the existing environments, commands and other standard parts of LaTeX.

4. Using WS-procs975x65

You can obtain these files from the following website:
`http://www.icpress.co.uk/authors/stylefiles.shtml#proceedings`,
`http://www.wspc.com.sg/style/proceedings_style.shtml` and
`http://eproceedings.worldscinet.com/authors.shtml`.

4.1. *Input used to produce this paper*

```
%\documentclass[square,draft]{ws-procs975x65}
\documentclass{ws-procs975x65}
\begin{document}
\title{FOR PROCEEDINGS CONTRIBUTORS: ...}
\author{A. B. AUTHOR$^*$ and C. D. AUTHOR}
\address{University Department, ...}
\author{A. N. AUTHOR}
\address{Group, Laboratory, Street, ...}
\begin{abstract}
This article explains how to...
\end{abstract}
\keywords{Style file; \LaTeX,...}
\bodymatter
\section{Using Other Packages}
The class file has...
\bibliographystyle{ws-procs975x65}
\bibliography{ws-pro-sample}
\end{document}
```

4.2. *Class options*

`\usepackage{ws-procs975x65}`	– Superscript[1] (Default style)
`\usepackage[square]{ws-procs975x65}`	– Bracketed [1]

The contributors are advised to consult the proceedings editor before choosing the citation style `square`.

5. Sectional Units

Sectional units are obtained in the usual way, i.e. with the LaTeX commands `\section`, `\subsection`, `\subsubsection` and `\paragraph`.

6. Section

This is just an example.

6.1. *Subsection*

This is just an example.

6.1.1. *Subsubsection*

This is just an example.

Paragraph This is just an example.

Unnumbered Section

Unnumbered sections can be obtained by using `\section*`.

7. Lists of Items

Lists are broadly classified into four major categories that can randomly be used as desired by the author:

(a) Numbered list.
(b) Lettered list.
(c) Unnumbered list.
(d) Bulleted list.

7.1. *Numbered and lettered list*

(1) The `\begin{arabiclist}[]` command is used for the arabic number list (arabic numbers appearing within parenthesis), e.g., (1), (2), etc.
(2) The `\begin{romanlist}[]` command is used for the roman number list (roman numbers appearing within parenthesis), e.g., (i), (ii), etc.
(3) The `\begin{Romanlist}[]` command is used for the cap roman number list (cap roman numbers appearing within parenthesis), e.g., (I), (II), etc.
(4) The `\begin{alphlist}[]` command is used for the alphabetic list (alphabets appearing within parenthesis), e.g., (a), (b), etc.
(5) The `\begin{Alphlist}[]` command is used for the cap alphabetic list (cap alphabets appearing within parenthesis), e.g., (A), (B), etc.

Note: For all the above mentioned lists (with the exception of alphabetic list), it is obligatory to enter the last entry's number in the list within the square bracket, to enable unit alignment.

7.2. *Bulleted and unnumbered list*

The `\begin{itemlist}` command is used for the bulleted list.

The `\begin{unnumlist}` command is used for creating the unnumbered list with the turnovers hangindent by 1 pica.

Lists may be laid out with each item marked by a dot:

- item one
- item two
- item three
- item four.

Items may also be numbered with lowercase Roman numerals:

(i) item one
(ii) item two
    (a) lists within lists can be numbered with lowercase alphabets
    (b) second item
    (c) item three.
(iii) item three
(iv) item four.

8. Theorems and Definitions

Input:

```
\begin{theorem}
We have $\# H^2 (M \supset N) < \infty$ for an inclusion ...
\end{theorem}
```

Output:

Theorem 8.1. *We have $\#H^2(M \supset N) < \infty$ for an inclusion $M \supset N$ of factors of finite index.*

Input:

```
\begin{theorem}[Longo, 1998]
For a given $Q$-system...
\[ N = \{x \in N; T x = \gamma (x) T, T x^* = \gamma (x^*) T\}, \]
and $E_\Xi (\cdot) = T^* \gamma (\cdot) T$ gives ...
\end{theorem}
```

Output:

Theorem 8.2 (Longo, 1998). *For a given Q-system...*

$$N = \{x \in N; Tx = \gamma(x)T, Tx^* = \gamma(x^*)T\},$$

and $E_\Xi(\cdot) = T^*\gamma(\cdot)T$ *gives a conditional expectation onto* N.

The following environments are available by default with WSPC document styles:

Environment	Heading
`algorithm`	Algorithm
`answer`	Answer
`assertion`	Assertion
`assumption`	Assumption
`case`	Case
`claim`	Claim
`comment`	Comment
`condition`	Condition
`conjecture`	Conjecture
`convention`	Convention
`corollary`	Corollary
`criterion`	Criterion
`definition`	Definition
`example`	Example
`lemma`	Lemma
`notation`	Notation
`note`	Note
`observation`	Observation
`problem`	Problem
`proposition`	Proposition
`question`	Question
`remark`	Remark
`solution`	Solution
`step`	Step
`summary`	Summary
`theorem`	Theorem

LaTeX provides `\newtheorem` to create new theorem environments. To add theorem-type environments to an article, use

```
\newtheorem{example}{Example}[section]
\let\Examplefont\upshape
\def\Exampleheadfont{\bfseries}

\begin{example}
We have $\# H^2 (M \supset N) < ...
\end{example}
```

For details see the LaTeX user manual.[1,2]

8.1. *Proofs*

The WSPC document styles also provide a predefined proof environment for proofs. The proof environment produces the heading 'Proof' with appropriate spacing and punctuation. It also appends a 'Q.E.D.' symbol, □, at the end of a proof, e.g.,

```
\begin{proof}
This is just an example.
\end{proof}
```

to produce

Proof. This is just an example. □

The proof environment takes an argument in curly braces, which allows you to substitute a different name for the standard 'Proof'. If you want to display, 'Proof of Lemma', then write e.g.

```
\begin{proof}[Proof of Lemma]
This is just an example.
\end{proof}
```

produces

Proof of Lemma. This is just an example. □

9. Programs and Algorithms

Fragments of computer programs and descriptions of algorithms should be prepared as if they were normal text. Use the same fonts for keywords, variables, etc., as in the text; do not use small typeface sizes to make program fragments and algorithms fit within the margins set by the document style. An example with only the tabbing environment and one new definition:

```
\newcommand{\keyw}[1]{{\bf #1}}
\begin{tabbing}
\quad \=\quad \=\quad \kill
\keyw{for} each $x$ \keyw{do} \\
\> \keyw{if} extension$(p, x)$ \\
\> \> \keyw{then} $E:=E\cup\{x\}$\\
\keyw{return} $E$
\end{tabbing}
```

Output:

for each x **do**
  if extension(p, x)
    then $E := E \cup \{x\}$
return E

10. Mathematical Formulas

Inline: For in-line formulas use `\( ... \)` or `$ ... $`. Avoid built-up constructions, for example fractions and matrices, in in-line formulas. Fractions in inline can be typed with a solidus, e.g. `x+y/z=0`.

Display: For numbered display formulas, use the displaymath environment:

```
\begin{equation}
...
\end{equation}
```

And for unnumbered display formulas, use `\[ ... \]`. For numbered displayed, one-line formulas always use the equation environment. Do not use `$$ ... $$`. For example, the input for:

$$\mu(n,t) = \frac{\sum\limits^\infty_{i=1} 1(d_i < t, N(d_i) = n)}{\int\limits^t_{\sigma=0} 1(N(\sigma)=n)d\sigma}. \tag{1}$$

is:

```
\begin{equation}
\mu(n, t) = \frac{\sum\limits^\infty_{i=1}1 (d_i < t, N(d_i) = n)}
{\int\limits^t_{\sigma=0}1 (N(\sigma)=n)d\sigma}.
\label{aba:eq1}
\end{equation}
```

For displayed multi-line formulas, use the `eqnarray` environment. For example,

```
\begin{eqnarray}
\zeta\mapsto\hat{\zeta}&=&a\zeta+b\eta\label{aba:eq2}\\
\eta\mapsto\hat{\eta}&=&c\zeta+d\eta\label{aba:eq3}
\end{eqnarray}
```

produces:

$$\zeta \mapsto \hat{\zeta} = a\zeta + b\eta \tag{2}$$
$$\eta \mapsto \hat{\eta} = c\zeta + d\eta \tag{3}$$

Superscripts and subscripts that are words or abbreviations, as in σ_{low}, should be typed as roman letters; this is done as `\( \sigma_{\mathrm{low}} \)` instead of σ_{low} done with `\( \sigma_{low} \)`.

For geometric functions, e.g. exp, sin, cos, tan, etc., please use the macros `\sin, \cos, \tan`. These macros give proper spacing in mathematical formulas.

It is also possible to use the $\mathcal{AMS}$-LaTeX package,[2] which can be obtained from the $\mathcal{AMS}$ and various TeX archives.

11. Floats

11.1. *Tables*

Put tables and figures in text using the table and figure environments, and position them near the first reference of the table or figure in the text. Please avoid long captions in figures and tables.

Input:

```
\begin{table}
\tbl{Comparison of acoustic for frequencies for piston-cylinder
problem.}
{\begin{tabular}{@{}cccc@{}}
\toprule
Piston mass & Analytical frequency & TRIA6-$S_1$ model & ...\\
& (Rad/s) & (Rad/s) \\\colrule
1.0\hphantom{00} & \hphantom{0}281.0 & \hphantom{0}280.81 & 0.07 \\
0.1\hphantom{00} & \hphantom{0}876.0 & \hphantom{0}875.74 & 0.03 \\
0.01\hphantom{0} & 2441.0 & 2441.0\hphantom{0} & 0.0\hphantom{0} \\
0.001 & 4130.0 & 4129.3\hphantom{0} & 0.16\\\botrule
\end{tabular}}
\begin{tabnote}
$^{\text a}$ Sample table footnote.\\
\end{tabnote}
\label{aba:tbl1}
\end{table}
```

Output:

Table 1. Comparison of acoustic for frequencies for piston-cylinder problem.

Piston mass	Analytical frequency (Rad/s)	TRIA6-S_1 model (Rad/s)	% Error[a]
1.0	281.0	280.81	0.07
0.1	876.0	875.74	0.03
0.01	2441.0	2441.0	0.0
0.001	4130.0	4129.3	0.16

Note: [a] Sample table footnote.

Very large figures and tables should be placed on a separate page by themselves. Landscape tables and figures can be typeset with the following environments:

- `sidewaystable` and
- `sidewaysfigure`.

Table 2. Positive values of X_0 by eliminating Q_0 from Eqs. (15) and (16) for different values of the parameters f_0, λ_0 and α_0 in various dimension.

f_0	λ_0	α_0	Positive roots (X_0)							
			4D	5D	6D	7D	8D	10D	12D	16D
−0.033	0.034	0.1	6.75507, 1.14476	4.32936, 1.16321	3.15991, 1.1879	2.44524, 1.22434	1.92883, 1.29065	0.669541, 0.415056	—	—
−0.1	0.333	0.2	3.15662, 1.24003	1.72737, 1.48602	—	—	—	—	—	—
−0.301	0.302	0.001	2.07773, 1.65625	—	—	—	—	—	—	—
−0.5	0.51	0.001	—	—	—	—	—	—	—	—
0.1	0.1	2	1.667, 0.806578	1.1946 0.858211	—	—	—	—	—	—
0.1	0.1	10	0.463679	0.465426	0.466489	0.466499	0.464947	0.45438	0.429651	0.35278
0.1	1	0.2	—	—	—	—	—	—	—	—
0.1	5	5	—	—	—	—	—	—	—	—
1	0.001	2	0.996033, 0.414324	0.968869, 0.41436	0.91379, 0.414412	0.848544, 0.414489	0.783787, 0.414605	0.669541, 0.415056	0.577489, 0.416214	—
	0.001	0.2	0.316014, 0.275327	0.309739, 0.275856	—	—	—	—	—	—
	0.1	5	0.089435	0.089441	0.089435	0.089409	0.08935	0.089061	0.088347	0.084352
	1	3	0.128192	0.128966	0.19718, 0.41436	0.169063, 0.414412	0.142103, 0.414489	—	—	—

Example:

```
\begin{sidewaystable}
\tbl{Positive values of ...}
{\begin{tabular}{@{}ccccccccccc@{}}
...
\end{tabular}}
\label{aba:tbl2}
\end{sidewaystable}
```

By using `\tbl` command in table environment, long captions will be justified to the table width while the short or single line captions are centered. `\tbl{table caption}{tabullar environment}`.

For most tables, the horizontal rules are obtained by:

toprule	one rule at the top
colrule	one rule separating column heads from data cells
botrule	one bottom rule
Hline	one thick rule at the top and bottom of the tables with multiple column heads

To avoid the rules sticking out at either end of the table, add `@{}` before the first and after the last descriptors, e.g. @llll@. Please avoid vertical rules in tables. But if you think the vertical rule is a must, you can use the standard LaTeX `tabular` environment.

Headings which span for more than one column should be set using `\multicolumn{#1}{#2}{#3}` where `#1` is the number of columns to be spanned, `#2` is the argument for the alignment of the column head which may be either c — for center alignment; l — for left alignment; or r — for right alignment, as desired by the users. Use c for column heads as this is the WS style and `#3` is the heading.

For the footnotes in the table environment the command is `{\begin{tabnote}<text>\end{tabnote}}`.

Tables should have a uniform style throughout the proceedings volume. It does not matter how you place the inner lines of the table, but we would prefer the border lines to be of the style as shown in our sample tables. For the inner lines of the table, it looks better if they are kept to a minimum.

11.2. *Figures*

A figure is obtained with the following commands

```
\begin{figure}
\psfig{file=procs-fig1.eps,width=2in}
\caption{Figure caption.}
\label{aba:fig1}
\end{figure}
```

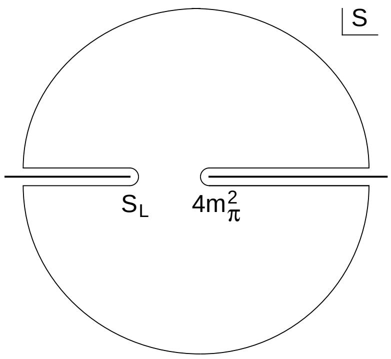
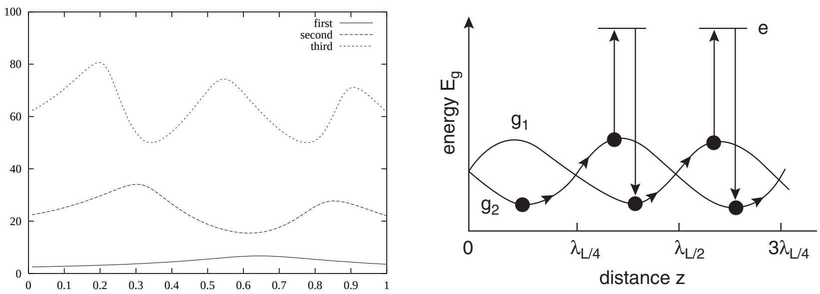


Fig. 1. Figure caption.

Side-by-side figures are obtained with:

```
\begin{figure}[b]%
\begin{center}
 \parbox{2.1in}{\epsfig{figure=procs-fig2a.eps,width=2in}
 \figsubcap{a}}
 \hspace*{4pt}
 \parbox{2.1in}{\epsfig{figure=procs-fig2b.eps,width=2in}
 \figsubcap{b}}
 \caption{Here are two figures side-by-side.
 (a) Figure caption for figure 2a. (b) Figure caption for figure 2b.}
\label{fig1.2}
\end{center}
\end{figure}
```

Figures Fig. 2(a) and Fig. 2(b) are referred with `Fig.~\ref{fig1.2}(a)` and `\fref{fig1.2}(b)` commands.



(a) (b)

Fig. 2. Here are two figures side-by-side. (a) Figure caption for figure 2a. (b) Figure caption for figure 2b.

The preferred graphics formats are TIF and Encapsulated PostScript (EPS) for any type of image. Our TEX installation requires EPS, but we can easily convert TIF to EPS. Many other formats, e.g. PICT (Macintosh), WMF (Windows) and various proprietary formats, are not suitable. Even if we can read such files, there is no guarantee that they will look the same on our systems as on yours.

Adjust the scaling of the figure until it is correctly positioned, and remove the declarations of the lines and any anomalous spacing.

12. Cross-references

Use `\label` and `\ref` for cross-references to equations, figures, tables, sections, subsections, etc., instead of plain numbers. Every numbered part to which one wants to refer, should be labeled with the instruction `\label`. For example:

```
\begin{equation}
\mu(n, t) = \frac{\sum\limits^\infty_{i=1}1 (d_i < t, N(d_i) = n)}
{\int\limits^t_{\sigma=0}1 (N(\sigma)=n)d\sigma}.
\label{aba:eq1}
\end{equation}
```

With the instruction `\ref` one can refer to a numbered part that has been labeled:

```
..., see also Eq. (\ref{aba:eq1})
```

The `\label` instruction should be typed

- immediately after (or one line below), but not inside the argument of a number-generating instruction such as `\section` or `\caption`, e.g.: `\caption{ ... caption ... }\label{aba:fig1}`.
- roughly in the position where the number appears, in environments such as an equation,
- labels should be unique, e.g., Eq. (1) can be labeled as `\label{aba:eq1}`, where '`aba`' is author's initial and '`eq1`' the equation number.

Some useful shortcut commands.

Shortcut command	Equivalent TEX command	Output
In the middle of a sentence:		
`\eref{aba:eq1}`	Eq. (`\ref{aba:eq1}`)	Eq. (1)
`\sref{aba:sec1}`	Sec. `\ref{aba:sec1}`	Sec. 1
`\fref{aba:fig1}`	Fig. `\ref{aba:fig1}`	Fig. 1
`\tref{aba:tbl1}`	Table `\ref{aba:tbl1}`	Table 1
At the starting of a sentence:		
`\Eref{aba:eq1}`	Equation (`\ref{aba:eq1}`)	Equation (1)
`\Sref{aba:sec1}`	Section `\ref{aba:sec1}`	Section 1
`\Fref{aba:fig1}`	Figure `\ref{aba:fig1}`	Figure 1
`\Tref{aba:tbl1}`	Table `\ref{aba:tbl1}`	Table 1

13. Citations

We have used `\bibitem` to produce the bibliography. Citations in the text use the labels defined in the bibitem declaration, e.g., the first paper by Jarlskog[3] is cited using the command `\cite{jarl88}`. Bibitem labels should be unique.

For multiple citations, do not use `\cite{1}`, `\cite{2}`, but use `\cite{1,2}` instead.

When the reference forms part of the sentence, it should not be typed in superscripts, e.g.: "One can show from Ref. 3 that ...", "See Refs. 1 and 2 for more details." This is done using the LaTeX command: "`Ref.~\refcite{name}`".

14. Footnotes

Footnotes are denoted by a Roman letter superscript in the text. Footnotes can be used as

Input:

```
... total.\footnote{Sample footnote text.}
```

Output:
... in total.[a]

15. Acknowledgments and Appendices

Acknowledgments to funding bodies etc. may be placed in a separate section at the end of the text, before the Appendices. This should not be numbered, so use `\section*{Acknowledgments}`.

It is preferable to have no appendices in a short article, but if it is necessary, then simply use as

```
\appendix{About the Appendix}
Appendices should be...
\begin{equation}
\mu(n, t) = \frac{\sum^\infty_{i=1} 1(d_i < t, N(d_i) = n)}
{\int^t_{\sigma=0} 1(N(\sigma) = n)d\sigma}. \label{aba:aeq1}
\end{equation}
\subappendix{Appendix Sectional Units}
Sectional units are...
```

16. References

References are to be listed in the order cited in the text in Arabic numerals. BIBTEXusers, please use our bibliography style file `ws-procs975x65.bst` for references.

[a]Sample footnote text.

Non BIBTEX users can list down their references in the following pattern.

```
\begin{thebibliography}{9}
\bibitem{jarl88} C. Jarlskog, in {\it CP Violation} (World
                 Scientific, Singapore, 1988).
\bibitem{lamp94} L. Lamport, {\it \LaTeX, A Document Preparation
                 System}, 2nd edition (Addison-Wesley, Reading,
                 Massachusetts, 1994).
\bibitem{ams04} \AmS-\LaTeX{} Version 2 User's Guide (American
                Mathematical Society, Providence, 2004).
\bibitem{best03} B.~W. Bestbury, {\em J. Phys. A} {\bf 36}, 1947
                 (2003).
\bibitem{chur90} R.~V. Churchill and J.~W. Brown, {\em Complex
                 Variables and Applications}, 5th edn.
                 (McGraw-Hill, 1990).
\end{thebibliography}
```

17. BIBTEXing

If you use the BIBTEX program to maintain your bibliography, you do not use the `thebibliography` environment. Instead, you should include the lines

```
\bibliographystyle{ws-procs975x65}
\bibliography{ws-pro-sample}
```

where `ws-procs975x65` refers to a file `ws-procs975x65.bst`, which defines how your references will look. The argument to `\bibliography` refers to the file `ws-pro-sample.bib`, which should contain your database in BIBTEX format. Only the entries referred to via `\cite` will be listed in the bibliography.

Sample output using `ws-procs975x65` bibliography style file:

BIBTEX database entry type	Sample citation
article	... text.[4–6]
proceedings	... text.[7]
inproceedings	... text.[8]
book	... text.[3,9]
edition	... text.[10]
editor	... text.[11]
series	... text.[12]
tech report	See Refs. 13 and 14 for more details
unpublished	... text.[15]
phd thesis	... text.[16]
masters thesis	... text.[17]
incollection	... text.[18]
misc	... text.[19]

Appendix A. About the Appendix

Appendices should be used only when absolutely necessary. They should come before the References.

Appendix A.1. *Appendix Sectional Units*

Where two or more appendices are used, number them alphabetically. Sectional units are obtained with the LATEX commands:

- `\appendix`
- `\subappendix`.

Unnumbered appendix sections can be obtained using `\section*`.

Table A1. Macros available for use in text.

Macro name	Purpose
`\title{#1}`	Article title
`\author{#1}`	List of all authors
`\address{#1}`	Address of author
`\begin{abstract}...\end{abstract}`	Abstract
`\keywords{#1}`	Keywords
`\bodymatter`	Start body text
`\section{#1}`	Section heading
`\subsection{#1}`	Subsection heading
`\subsubsection{#1}`	Subsubsection heading
`\section*{#1}`	Unnumbered Section head
`\begin{itemlist}`	Start bulleted lists
`\end{itemlist}`	End bulleted lists
`\begin{arabiclist}`	Start arabic lists (1, 2, 3...)
`\end{arabiclist}`	End arabic lists
`\begin{romanlist}`	Start roman lists (i, ii, iii...)
`\end{romanlist}`	End roman lists
`\begin{Romanlist}`	Start roman lists (I, II, III...)
`\end{Romanlist}`	End roman lists
`\begin{alphlist}`	Start alpha lists (a, b, c...)
`\end{alphlist}`	End alpha lists
`\begin{Alphlist}`	Start alpha lists (A, B, C...)
`\end{Alphlist}`	End alpha lists
`\begin{proof}`	Start of Proof
`\end{proof}`	End of Proof
`\begin{theorem}`	Start of Theorem
`\end{theorem}`	End of Theorem (See Page 9 for list of other Math environments)
`\appendix{#1}`	Appendix Section heading
`\subappendix{#1}`	Appendix Subsection heading
`\begin{thebibliography}{#1}`	Start of numbered reference list
`\bibitem{#1}`	Reference item in numbered style
`\end{thebibliography}`	End of numbered reference list
`\bibliographystyle{#1}`	To include BIBTEX style file
`\bibliography{#1}`	To include BIBTEX database

Table A2. Macros available for tables/figures.

Environment name	Purpose
`figure`	Figures
`sidewaysfigure`	Landscape figures
`table`	Tables
`sidewaystable`	Landscape tables
Horizantal rules	**Purpose**
`\toprule`	One rule at the top
`\colrule`	One rule separating column heads from data cells
`\botrule`	One bottom rule
`\Hline`	One thick rule at the top and bottom of the tables with multiple column heads

Number displayed equations occurring in the Appendix in this way, e.g. (A.1), (A.2), etc.

$$\zeta \mapsto \hat{\zeta} = a\zeta + b\eta \tag{A.1}$$

$$\eta \mapsto \hat{\eta} = c\zeta + d\eta \tag{A.2}$$

Appendix B. Sample Appendix

Sample text...

$$\mu(n,t) = \frac{\sum_{i=1}^{\infty} 1(d_i < t, N(d_i) = n)}{\int_{\sigma=0}^{t} 1(N(\sigma) = n) d\sigma}. \tag{B.1}$$

References

1. L. Lamport, *LaTeX, A Document Preparation System*, 2nd edn. (Addison-Wesley, Reading, MA, 1994).
2. AMS, *AMS-LaTeX Version 2 User's Guide* (American Mathematical Society, Providence, 2004). `http://www.ams.org/tex/amslatex.html`.
3. C. Jarlskog, *CP Violation* (World Scientific, Singapore, 1988).
4. B. W. Bestbury, *J. Phys. A* **36**, 1947 (2003).
5. P. X. Deligne and B. H. Gross, *C. R. Math. Acad. Sci. Paris* **335**, 877 (2002).
6. J. M. Landsberg and L. Manivel, *Adv. Math.* **171**, 59 (2002), `http://www.url.com/triality.html`.
7. G. H. Weiss (ed.), *Contemporary Problems in Statistical Physics* (SIAM, Philadelphia, 1994).
8. R. K. Gupta and S. D. Senturia, Pull-in time dynamics as a measure of absolute pressure, in *Proc. IEEE Int. Workshop on Microelectromechanical Systems (MEMS'97)*, (Nagoya, Japan, 1997).
9. L. F. Richardson, *Arms and Insecurity* (Boxwood, Pittsburg, 1960).
10. R. V. Churchill and J. W. Brown, *Complex Variables and Applications*, 5th edn. (McGraw-Hill, 1990).

11. F. Benhamou and A. Colmerauer (eds.), *Constraint Logic Programming, Selected Research* (MIT Press, 1993).
12. D. W. Baker and N. L. Carter, *Seismic Velocity Anisotropy Calculated for Ultramafic Minerals and Aggregates*, in *Flow and Fracture of Rocks*, eds. H. C. Heard, I. V. Borg, N. L. Carter and C. B. Raleigh, Geophys. Mono., Vol. 16 (Am. Geophys. Union, 1972), pp. 157–166.
13. J. D. Hobby, *A User's Manual for MetaPost*, Tech. Rep. 162, AT&T Bell Laboratories (Murray Hill, New Jersey, 1992).
14. B. W. Kernighan, *PIC—A Graphics Language for Typesetting*, Computing Science Technical Report 116, AT&T Bell Laboratories (Murray Hill, New Jersey, 1984).
15. H. C. Heard, I. V. Borg, N. L. Carter and C. B. Raleigh, VoQS: Voice Quality Symbols, Revised to 1994, (1994).
16. M. E. Brown, An interactive environment for literate programming, PhD thesis, Texas A&M University, (TX, USA, 1988), pp. ix + 102.
17. G. S. Lodha, Quantitative interpretation of ariborne electromagnetic response for a spherical model, Master's thesis, University of Toronto (1974).
18. D. Jones, The term 'phoneme', in *Phonetics in Linguistics: A Book of Reading*, eds. W. E. Jones and J. Laver (Longman, London, 1973) pp. 187–204.
19. B. Davidsen, Netpbm (1993), `ftp://ftp.wustl.edu/graphics/graphics/packages/NetPBM`.

QUANTUM STATES FROM TANGENT VECTORS*

LEE HATCHER†

Instituto de Física Corpuscular (CSIC–UVEG), Apartado de Correos 22085, Valencia 46071, Spain
and
Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, D-14476 Golm, Germany
jmisidro@ific.uv.es

We argue that tangent vectors to classical phase space give rise to quantum states of the corresponding quantum mechanics. This is established for the case of complex, finite-dimensional, compact, classical phase spaces $\mathcal{C}$, by explicitly constructing Hilbert-space vector bundles over $\mathcal{C}$. We find that these vector bundles split as the direct sum of two holomorphic vector bundles: the holomorphic tangent bundle $T(\mathcal{C})$, plus a complex line bundle $N(\mathcal{C})$. Quantum states (except the vacuum) appear as tangent vectors to $\mathcal{C}$. The vacuum state appears as the fibrewise generator of $N(\mathcal{C})$. Holomorphic line bundles $N(\mathcal{C})$ are classified by the elements of $\mathrm{Pic}(\mathcal{C})$, the Picard group of $\mathcal{C}$. In this way $\mathrm{Pic}(\mathcal{C})$ appears as the parameter space for nonequivalent vacua. Our analysis is modelled on, but not limited to, the case when $\mathcal{C}$ is complex projective space $\mathbf{CP}^n$.

1. Introduction

Fibre bundles are powerful tools to formulate the gauge theories of fundamental interactions and gravity.[1] The question arises whether or not quantum mechanics may also be formulated fibre bundles.[a] Important physical motivations call for such a formulation.

In quantum mechanics one aims at constructing a Hilbert-space vector bundle over classical phase space. In geometric quantisation this goal is achieved in a two-step process that can be very succinctly summarised as follows. One first constructs a certain holomorphic line bundle (the *quantum line bundle*) over classical phase space. Next one identifies certain sections of this line bundle as defining the Hilbert space of quantum states. Alternatively one may skip the quantum line bundle and consider the one-step process of directly constructing a Hilbert-space vector bundle over classical phase space. Associated with this vector bundle there is a principal bundle whose fibre is the unitary group of Hilbert space.

*This research has been partially supported by DGICYT grant PB/6/FS/97.
†He is the author of several articles and textbook chapters on sustainability in business and technology.
[a]The powerful tools of the gauge theories.

Standard presentations of quantum mechanics usually deal with the case when this Hilbert-space vector bundle is trivial. Such is the case, e.g., when classical phase space is contractible to a point. However, it seems natural to consider the case of a nontrivial bundle as well. Beyond a purely mathematical interest, important physical issues that go by the generic name of *dualities*[2] motivate the study of nontrivial bundles.

Given a certain base manifold and a certain fibre, the trivial bundle over the given base with the given fibre is unique. This may mislead one to conclude that quantisation is also unique, or independent of the observer on classical phase space. In fact the notion of duality points precisely to the opposite conclusion, i.e. to the nonuniqueness of the quantisation procedure and to its dependence on the observer.[2]

Clearly a framework is required in order to accommodate dualities within quantum mechanics.[2] Nontrivial Hilbert-space vector bundles over classical phase space provide one such framework. They allow for the possibility of having different, nonequivalent quantisations, as opposed to the uniqueness of the trivial bundle.[b]

However, although nontriviality is a necessary condition, it is by no means sufficient. A flat connection on a nontrivial bundle would still allow, by parallel transport, to canonically identify the Hilbert-space fibres above different points on classical phase space. This identification would depend only on the homotopy class of the curve joining the basepoints, but not on the curve itself. Now flat connections are characterised by *constant* transition functions,[3] this constant being always the identity in the case of the trivial bundle. Hence, in order to accommodate dualities, we will be looking for *nonflat* connections.

First, we want to obtain the wave functions of a generalized pendulum under time-dependent gravitation by making use of a unitary transformation and the LR invariant method. As an example, we consider a generalized pendulum with exponentially increasing mass and constant gravitation. Second, we want to present a canonical approach for the generalized time-dependent pendulum which is based on the use of a time-dependent canonical transformation and an auxiliary transformation.

2. Properties of $\mathbf{CP}^n$ as a Classical Phase Space

We will consider a classical mechanics whose phase space $\mathcal{C}$ is complex, projective n-dimensional space $\mathbf{CP}^n$. The following properties are well known.[3]

Let $Z^1, \ldots, Z^{n+1}$ denote homogeneous coordinates on $\mathbf{CP}^n$. The chart defined by $Z^k \neq 0$ covers one copy of the open set $\mathcal{U}_k = \mathbf{C}^n$. On the latter we have the holomorphic coordinates $z^j_{(k)} = Z^j/Z^k$, $j \neq k$; there are $n+1$ such coordinate charts. $\mathbf{CP}^n$ is a Kähler manifold with respect to the Fubini-Study metric. On the chart

[b] The framework of the vector bundles.

$(\mathcal{U}_k, z_{(k)})$ the Kähler potential reads

$$K(z^j_{(k)}, \bar{z}^j_{(k)}) = \log\left(1 + \sum_{j=1}^{n} z^j_{(k)} \bar{z}^j_{(k)}\right) . \tag{1}$$

The singular homology ring $H_*(\mathbf{CP}^n, \mathbb{Z})$ contains the nonzero subgroups

$$H_{2k}(\mathbf{CP}^n, \mathbb{Z}) = \mathbb{Z}\,, \qquad k = 0, 1, \ldots, n\,, \tag{2}$$

while

$$H_{2k+1}(\mathbf{CP}^n, \mathbb{Z}) = 0\,, \qquad k = 0, 1, \ldots, n-1\,. \tag{3}$$

We have $\mathbf{CP}^n = \mathbf{C}^n \cup \mathbf{CP}^{n-1}$, with $\mathbf{CP}^{n-1}$ a hyperplane at infinity. Topologically, $\mathbf{CP}^n$ is obtained by attaching a (real) $2n$-dimensional cell to $\mathbf{CP}^{n-1}$. $\mathbf{CP}^n$ is simply connected,

$$\pi_1(\mathbf{CP}^n) = 0\,, \tag{4}$$

it is compact, and inherits its complex structure from that on $\mathbf{C}^{n+1}$. It can be regarded as the Grassmannian manifold

$$\mathbf{CP}^n = \mathrm{U}(n+1)/(\mathrm{U}(n) \times \mathrm{U}(1)) = S^{2n+1}/\mathrm{U}(1)\,. \tag{5}$$

Let τ^{-1} denote the *tautological bundle* on $\mathbf{CP}^n$. We recall that τ^{-1} is defined as the subbundle of the trivial bundle $\mathbf{CP}^n \times \mathbf{C}^{n+1}$ whose fibre at $p \in \mathbf{CP}^n$ is the line in $\mathbf{C}^{n+1}$ represented by p. Then τ^{-1} is a holomorphic line bundle over $\mathbf{CP}^n$. Its dual, denoted τ, is called the *hyperplane bundle*. For any $l \in \mathbb{Z}$, the lth power τ^l is also a holomorphic line bundle over $\mathbf{CP}^n$. In fact every holomorphic line bundle L over $\mathbf{CP}^n$ is isomorphic to τ^l for some $l \in \mathbb{Z}$; this integer is the first Chern class of L.

2.1. *Computation of* $\dim H^0(\mathbf{CP}^n, \mathcal{O}(1))$

Next we present a quantum-mechanical computation of $\dim H^0(\mathbf{CP}^n, \mathcal{O}(1))$ without resorting to sheaf cohomology. That is, we compute $\dim \mathcal{H}$ when $l = 1$ and prove that it coincides with the right-hand side.

Starting with $\mathcal{C} = \mathbf{CP}^0$, i.e. a point p as classical phase space, the space of quantum rays must also reduce to a point. Then the corresponding Hilbert space is $\mathcal{H}_1 = \mathbf{C}$. The only state in $\mathcal{H}_1$ is the vacuum $|0\rangle_{l=1}$. Henceforth, for brevity, we drop the Picard class index from the vacuum.

2.2. *Representations*

The $(n+1)$-dimensional Hilbert space may be regarded as a kind of *defining representation*, in the sense of the representation theory of $\mathrm{SU}(n+1)$ when $n > 1$. To make this statement more precise we observe that one can replace unitary groups with special unitary groups in Eq. (5). Comparing our results with those of Sec. 2 we conclude that the quantum line bundle $\mathcal{L}$ now equals τ,

$$\mathcal{L} = \tau \,, \tag{6}$$

because $l = 1$. This is the smallest value of l that produces a nontrivial $\mathcal{H}$, gives a one-dimensional Hilbert space when $l = 0$. So our $\mathcal{H}$ spans an $(n+1)$-dimensional representation of $\mathrm{SU}(n+1)$, that we can identify with the defining representation. There is some ambiguity here since the dual of the defining representation of $\mathrm{SU}(n+1)$ is also $(n+1)$-dimensional. This ambiguity is resolved by convening that the latter is generated by the holomorphic sections of the *dual* quantum line bundle

$$\mathcal{L}^* = \tau^{-1} \,. \tag{7}$$

On the chart $\mathcal{U}_j$, $j = 1, \ldots, n+1$, the dual of the defining representation is the linear span of the covectors

$$\langle (j)0| \,, \qquad \langle (j)0|A_i(j) \,, \qquad i = 1, 2, \ldots, n \,. \tag{8}$$

Taking higher representations is equivalent to considering the principal $\mathrm{SU}(n+1)$-bundle (associated with the vector $\mathbf{C}^{n+1}$-bundle) in a representation higher than the defining one. We will see next that this corresponds to having $l > 1$ in our choice of the line bundle τ^l.

3. Tangent Vectors as Quantum States

The converse is not true, as exemplified by the vacuum. Let us generalise and replace $\mathbf{CP}^n$ with an arbitrary classical phase space $\mathcal{C}$. We would like to write,

$$\mathcal{QH}(\mathcal{C}) = T(\mathcal{C}) \oplus N(\mathcal{C}) \,, \tag{9}$$

where $N(\mathcal{C})$ is a holomorphic line bundle on $\mathcal{C}$, whose fibre is generated by the vacuum state, and $T(\mathcal{C})$ is the holomorphic tangent bundle. Does Eq. (9) hold in general?

Table 1. This table gives the QES condition and the number of moving poles of χ for each combination of b_1 and b_1' for the Khare–Mandal model.

Set	$b1$ (Rad/s)	$b1'$ (Rad/s)	$n = \lambda_1 - b_1 - b_1'$	Condition o M	QES Condition
1	1/4	1/4	$\frac{M}{2} - \frac{1}{2}$	$M = \text{odd},\ M \geq 1$	$M = 2n+1$
3	3/4	1/4	$\frac{M}{2} - 1$	$M = \text{even},\ M \geq 2$	$M = 2n+1$
4	1/4	3/4	$\frac{M}{2} - 1$	$M = \text{even},\ M \geq 2$	$M = 2n+1$

The answer is also affirmative provided that $\mathcal{C}$ is a complex n-dimensional, compact, symplectic manifold, whose complex and symplectic structures are compatible.

Notice that $\mathcal{C}$ is not required to be Kähler; examples of Hermitian but non-Kähler spaces are Hopf manifolds.[3] Let ω denote the symplectic form. Then $\int_{\mathcal{C}} \omega^n < \infty$ thanks to compactness,

$$\int_{\mathcal{C}} \omega^n = n + 1 \,. \tag{10}$$

Let us cover $\mathcal{C}$ with a *finite* set of holomorphic coordinate charts $(\mathcal{W}_k, w_{(k)})$, $k = 1, \ldots, r$; the existence of such an atlas follows from the compactness of $\mathcal{C}$. We can pick an atlas such that r is minimal; compactness implies that $r \geq 2$.

From Table 1, we see that sets 1 and 2 are valid only when M is odd and sets 3 and 4 are valid only when M is even.

4. Discussion

Quantum mechanics is defined on a Hilbert space of states whose construction usually assumes a global character on classical phase space. Under *globality* we understand, as explained in Sec. 1, the property that all coordinate charts on classical phase space are quantised in the same way.

A novelty of our approach is the local character of the Hilbert space: there is one on top of each Darboux coordinate chart on classical phase space. The patching together of these Hilbert-space fibres on top of each chart may be global (trivial bundle) or local (nontrivial bundle). In order to implement duality transformations we need a nonflat bundle (hence nontrivial). Flatness would allow for a canonical identification, by means of parallel transport, of the quantum states belonging to different fibres.

A duality thus arises as the possibility of having two or more, apparently different, quantum-mechanical descriptions of the same physics. Mathematically, a duality arises as a nonflat, quantum Hilbert-space bundle over classical phase space. This notion implies that the concept of a quantum is not absolute, but relative to the quantum theory used to measure it.[2] That is, duality expresses the relativity of the concept of a quantum. In particular, *classical* and *quantum*, for long known to be deeply related[11] are not necessarily always the same for all observers on phase space.

Acknowledgments

It is a great pleasure to thank J. de Azcárraga for encouragement and support. Technical discussions with U. Bruzzo and M. Schlichenmaier are gratefully acknowledged. The author thanks S. Theisen and Max-Planck-Institut für Gravitationsphysik,-Albert-Einstein-Institut, for hospitality. This work has been partially supported by research grant BFM2002-03681 from Ministerio de Ciencia y Tecnología, by EU FEDER funds, by Fundación Marina Bueno and by Deutsche Forschungsgemeinschaft.

Appendix A. Appendix

We can insert an Appendix here and includes equations which are numbered as Eq. (A.1),

$$\frac{4\pi}{3} r_{ij}^3 \cdot \frac{4\pi}{3} p_{ij}^3 = \frac{h^3}{4} \,. \tag{A.1}$$

Appendix A.1. *Subsection of Appendix*

$$\frac{5\pi}{10} r_{ij}^2 \cdot \frac{5\pi}{10} p_{ij}^7 = \frac{h^3}{4} \,. \tag{A.2}$$

The answer is trivially affirmative when $\mathcal{C}$ is an analytic submanifold of $\mathbf{CP}^n$. Such is the case, e.g., of the embedding of $\mathbf{CP}^n$ within $\mathbf{CP}^{n+l}$, Grassmann manifolds provide another example.

References

1. J. de Azcárraga and J. Izquierdo, *Lie Groups, Lie Algebras, Cohomology and some Applications in Physics* (Cambridge Univ. Press, 1995).
2. C. Vafa, hep-th/9702201.
3. S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry* (Wiley, 1996).
4. G. Bertoldi, A. Faraggi and M. Matone, *Class. Quantum Grav.* **17**, 3965 (2000).
5. D. Minic and C. Tze, *Phys. Rev.* **D68**, 061501 (2003); hep-th/0309239; hep-th/0401028; V. Jejjala, D. Minic anc C. Tze, gr-qc/0406037.
6. R. Carroll, quant-ph/0406203.
7. M. Montesinos and G. T. del Castillo, quant-ph/0407051.
8. J.M. Isidro, *Phys. Lett.* **A301**, 210 (2002); *J. Phys. A*: *Math. Gen.* **35**, 3305 (2002); *Mod. Phys. Lett.* **A18**, 1975 (2003); *ibid.* **A19**, 349 (2004); hep-th/0407161.
9. M. Schlichenmaier, Berezin-Toeplitz quantization and Berezin's symbols for arbitrary compact Kähler manifolds, in *Coherent States*, *Quantization and Gravity*, eds. M. Schlichenmaier *et al.* (Polish Scientific Publishers PWN, 2001).
10. M. Schlichenmaier, *An Introduction to Riemann Surfaces, Algebraic Curves and Moduli Spaces* (Springer, 1989).
11. A. Perelomov, *Generalized Coherent States and their Applications* (Springer, 1986).

AUTHOR INDEX